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Robert Berne and Leanna Stiefel

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*Robert Berne*  
*Leanna Stiefel*

## **The Equity of School Finance Systems Over Time: The Value Judgments Inherent in Evaluation<sup>1</sup>**

*Numerous value judgments are embedded in the standards utilized to evaluate whether a school finance system has become more or less equitable over time. A four-component framework is proposed to highlight these value judgments in equity standards. Data from twenty states are analyzed to demonstrate that alternative value judgments affect the measurement of a state's movement toward or away from equity. Implications for policymakers and administrators are discussed.*

*Robert Berne is an Assistant Professor of Public Administration and Leanna Stiefel is an Assistant Professor of Economics, both in the Graduate School of Public Administration, New York University.*

**A**nalyses of school finance systems utilize many different procedures to evaluate the equity of a state's system over time. For example, a recent issue of the *Journal of Education Finance*<sup>2</sup> was entirely devoted to reports of evaluations of twelve state systems that had recently undergone reform. No two of these twelve reports used the same methodology for evaluation.

If we were able to examine the universe of school finance evaluations, some of the differences in evaluation procedures could probably be attributed to the individual creativity of the analyst in presenting and organizing the data. Other differences could stem from the selection of a particular conception of equity as the standard against which to judge the state's school finance system. The choice of a standard may be based on the particular perspective of the group sponsoring the evaluation or on the analyst's interpretation of the intended effects of the school finance reform. But regardless of the reason for choosing one standard or another, the choice of a standard is too important to leave to interest groups or analysts. The selection of standards for equity evaluation needs to be widely discussed in the education community since legitimate disagreement over the choice of a standard is predictable, and different standards yield different results depending on the value judgments inherent in the standard.

Since school finance systems and reform proposals are complex, it is unlikely that standards of equity will be simple. The formulation of standards of equity involves a number of choices, and in this article we present a scheme to organize and simplify these choices and also to document that the choices have an impact on the results of the evaluations.

We begin by briefly highlighting, at a conceptual level, the value judgments inherent in different standards of equity. In this process, we describe a four-component framework for constructing equity standards. Second, we illustrate the kinds of conflicts that arise when alternative standards are employed to evaluate the equity of a state's school finance system intertemporally. The illustration is based on actual data from twenty states over a time period between 1970 and 1977. Third, we conclude with observations on the meaning of the existence of conflicts for policymakers and administrators.

## VALUE JUDGMENTS IN EQUITY STANDARDS

An evaluation of the equity of a school finance system must explicitly or implicitly begin with a definition of the standard of equity against which the finance system will be judged. Equity standards can be described by identifying the way they answer the four questions: who, what, how, and how much? These questions present evaluators with several choices for answers, each of which can be interpreted as a specification of value judgments about what is important. In this section, the kinds of choices that must be made to answer the four questions are described, and, in the process of the description, the value judgments inherent in alternative standards of equity as applied to school finance systems are identified. It should be emphasized that the description of alternative standards does not address the question of causes of inequity nor the question of how the choice for an answer to each of the four questions is determined. These are important questions, but too broad to be discussed in this article.

### Who? The Choice of a Group

The identification of an equity standard begins with the specification of the group of concern. In school finance evaluations, the choice has traditionally been between children and taxpayers. The choice of children is usually based on two rationales. Many people believe that the quality of education a child receives will have an important influence

on future life status or well-being. In order to make the distribution of future life status equitable, these people would argue that it is important that children's educational experiences be equitably distributed. A second rationale for the specification of children as a group depends not on the effect that the quality of education has on future status, but rather on a concern for the experiences of children while in school. It is sometimes argued that because children spend such a large part of their lives interacting with the education system, there is an obligation on the part of society to ensure that the quality of their experiences is provided equitably.

The second common choice for a group is the taxpayers. Because a public school finance system involves both a distribution of education resources among children and a distribution of tax liabilities among households, and also because during the 1970s the level of tax burden has become an issue of general concern, it is natural that some standards of equity would choose to emphasize the taxpayers rather than the children.<sup>3</sup>

In this article we concentrate on children as a group. The remaining three questions that need to be answered in order to establish an equity standard — what, how, and how much — are described from the children's viewpoint, and the empirical analysis is based on children as a group. The conclusions of the article, however, are equally relevant to taxpayers and to children.

### **What? The Choice of an Object to Be Distributed**

The choice of a group involves the question of equity for whom; the choice of an object involves the question of equity of what. Three alternatives available to answer the question "what" correspond to the stages of the educational process when it is viewed in an input-output context. First the evaluator can use educational inputs as the object. Inputs have been measured in a number of ways, including revenues or expenditures per child, cost adjusted revenues or expenditures per child, or real resources such as teachers or supplies per child. In the empirical section of this article, we use total state plus local revenues per child as the object.

A second possible choice for an object is the output of the education system, measured, for example, by achievement test scores or by years of schooling completed. There are many well-documented problems involved in obtaining reliable and valid measures of outputs, but conceptually they may still be considered the most important in an equity evaluation.

Finally, the outcomes that are related to the education system could be chosen for the object. Outcomes include such things as lifetime income, satisfaction, or status. If educational outputs such as achievement are deemed difficult to measure, then outcomes are even more so. But measurement problems do not negate the fact that some people's concern for educational equity is based on perceived relationships between education and the distribution of lifetime outcomes. Using outcomes as the object is a direct and explicit way of representing this viewpoint.

### **How? The Choice of an Equity Principle**

In order to assess the equity of a particular situation, the principles that specify how the object is distributed among the group members must be elaborated. A review of equity principles across a number of fields of study, including school finance, indicates that there are three alternatives: horizontal equity, vertical equity, and equal opportunity. Each of these is examined in turn, and all three are illustrated with an example.

The equal treatment of equals is a specification of the horizontal equity principle. If it is assumed that the group is composed of equally deserving members, then horizontal equity implies that each of these members should receive the same amount of the object. If the objects are not distributed equally across the group, then the degree of horizontal equity can be assessed by measuring the spread or dispersion of the distribution.

If the members of the group are not equal and some are judged to be deserving of more or less of the object of concern, then the application of the vertical rather than the horizontal equity principle may be appropriate. Vertical equity can be expressed as the unequal treatment of unequals. It is necessary to go beyond the identification of a group and object to specify the vertical equity principle. In addition, characteristics that identify more and less deserving members must be defined, and, in certain cases, the magnitude of the desired differential treatment must be determined.

Finally, there are certain situations where the equity principle can be expressed as the absence of discrimination. In this case, the concern is with equal opportunities for all members of the group. This principle differs from horizontal equity since particular members of the group are singled out because they are usually the object of discrimination. Minority group members and pupils in low-wealth districts are commonly thought to be denied equal opportunity. However, equal opportunity does not imply that these subgroups are more deserving, only that the subgroups should not systematically receive less of a particular object.

The three principles can be illustrated with an example in which the equity of the distribution of state and local revenues among children in a state is to be evaluated. It is assumed that the state is divided into school districts that vary both in their property wealth per child and in their proportions of children with learning handicaps.

Horizontal equity is concerned with the equality of revenues for all children. Children are assumed to be equally deserving, and horizontal equity measures the difference between the actual distribution and a perfectly equal distribution. The smaller the difference from equality, the greater the horizontal equity. In the next section, we discuss how dispersion measures such as the range of variation are possible measures of horizontal equity.

If learning handicaps are recognized as a characteristic that identifies children who should receive more revenues than children without learning handicaps, then the principle of vertical equity is applicable. Relationship-type measures can be used to assess the degree of vertical equity by examining the average revenues per child in districts with higher and lower proportions of learning handicapped children.<sup>4</sup> Vertical equity would require that districts with a higher proportion of learning handicapped children have higher revenues per child, and a more precise articulation could specify the magnitude of additional revenues that is desired.

Finally, the example can be used to illustrate the equal opportunity principle, since numerous state court cases have ruled that children's education should not depend on the property wealth of a child's school district.<sup>5</sup> Again, relationship measures can be utilized to determine whether higher wealth districts have higher revenues per child than do lower wealth districts. The equal opportunity principle requires that there be no systematic relationship between wealth per child and revenues per child.

The underlying premises and the actual formulation of the three equity principles — horizontal equity, vertical equity, and equal opportunity — indicate that the conclusions that are drawn from each principle are likely to be different. Therefore, it is critical that evaluation studies clearly identify those principles that have been selected and those that have been omitted.

### **How Much? The Choice of a Numerical Summary Measure**

The ideas of horizontal equity, vertical equity, and equal opportunity are broad equity principles. Their application in a specific study can be effected by a variety of dispersion or relationship measures. In the

previous section, a few specific measures were used to illustrate the broad principles. The range and the coefficient of variation were the dispersion measures cited in connection with the horizontal equity principle. The average revenues for groups of children arranged by proportions of learning disabled children or arranged by property values per child were the relationship measures used to illustrate the vertical equity and the equal opportunity principles. There are many other dispersion and relationship measures that could be used, and each measure implies certain value judgments. Much scholarly attention has been devoted to the identification of value judgments inherent in alternative summary measures of equity, and that literature is now drawn upon for an analysis of equity measures in school finance equity standards.<sup>6</sup>

*Univariate Dispersion Measures.* Studies of the equity of school finance systems commonly utilize one or more of nine dispersion measures to capture the degree to which horizontal equity has been achieved. These nine measures are listed and described below. The descriptions assume that we are concerned about the distribution of revenues per child across all children in a state. Where applicable, the mathematical formula for a measure is presented as well. The following symbols are used in the descriptions and formulas:

$P_i$  = Number of children (pupils) in district  $i$ .

$N$  = Number of districts in the state.

$R_i$  = Average revenue per child in district  $i$ .

$\bar{R}$  = Mean revenue per child for all children in the state.

$M$  = Median revenue per child for all children in the state.

1. The Range: The difference between the highest and lowest value of  $R_i$  in the distribution.
2. The Restricted Range: The difference between the value of  $R_i$  below which five percent of the children fall and the value of  $R_i$  above which five percent of the children fall.
3. The Federal Range Ratio: The restricted range divided by the value of  $R_i$  below which five percent of the children fall.
4. The Relative Mean Deviation: The ratio of the sum of the absolute value of the deviations of  $R_i$  from its mean to the sum of  $R_i$ .

$$\frac{\sum_{i=1}^N P_i \left| \bar{R} - R_i \right|}{\bar{R} \sum P_i}$$

5. **The Permissible Variance:** The ratio of the actual sum of the  $R_i$  for children below the median to the sum of  $R_i$  that would be required if all children were at the median level.

$$\frac{\sum_{i=1}^J P_i R_i}{M \sum_{i=1}^J P_i}$$

where districts 1 through J are below M.

6. **The Variance:** The average of the sum of the squared deviations of  $R_i$  from its mean.

$$\frac{\sum_{i=1}^N P_i (\bar{R} - R_i)^2}{\sum_{i=1}^N P_i}$$

7. **The Coefficient of Variation:** The square root of the variance divided by the mean.

$$\frac{\sqrt{\text{VARIANCE}}}{\bar{R}}$$

8. **The Standard Deviation of Logarithms:** The square root of the variance of the natural logarithms of  $R_i$ .

$$\left( \frac{\sum_{i=1}^N P_i (Z - \log_e R_i)^2}{\sum_{i=1}^N P_i} \right)^{1/2} \text{ where } Z = \frac{\sum_{i=1}^N P_i (\log_e R_i)}{\sum_{i=1}^N P_i}$$

9. **The Gini Coefficient:** The ratio of the area between the Lorenz curve and the 45° line to the area below the 45° line.

$$\frac{\sum_{i=1}^N \sum_{j=1}^N P_i P_j |R_i - R_j|}{2 \left( \sum_{i=1}^N P_i \right)^2 \bar{R}}$$

Each of the nine measures represents a different combination of value judgments about the degree of inequity that occurs when revenues are not exactly the same for each child. These value judgments can be represented by a series of questions. Figure 1 presents the answers to eight such questions. The questions are listed along the left side of the matrix in Figure 1 and the measures are displayed along the top of the matrix. In order to illustrate the meaning of these questions and answers, questions 2 and 4 have been selected for further discussion.

Value judgment 2 asks if the measure always shows an improvement in equity when revenues are transferred from any child to another who is lower in the distribution. Such a transfer does not change the relative positions of the children involved in the transfer, and it also maintains the mean revenue per child at the same level before and after the transfer. This type of transfer is often referred to as "mean preserving." Many people would think that such transfers should increase equity. Yet the first five measures listed across the top of Figure 1 do not register an increase (range, restricted range, federal range ratio, relative mean deviation, permissible variance). Therefore, the use of any of these five measures means the evaluator does not think that transfers of the kind described should always increase equity.

Question 4 asks if the measure always registers a change in equity when each child receives an equal absolute addition to revenues (for example, \$50 each). Such a change is sometimes seen as promoting equity because children with low original revenues benefit proportionally more than children with high original revenues. Three of the nine univariate measures do not register a change when equal additions are made (the range, the restricted range, the variance). Therefore, these three measures would not be consistent with a value judgment that such additions increase equity.

*Bivariate and Multivariate Relationship Measures.* Relationship measures can be represented by tables of average revenues for groups of children or districts arranged by property wealth per child or by proportions of learning handicapped, as described previously. While such tables are commonly used in exposition, conclusions in evaluation studies often rely more heavily on bivariate and multivariate summary measures such as correlations, slopes, and elasticities.

Seven commonly used bivariate or multivariate summary measures have been identified. The seven measures are listed below with an abbreviation and a description for each one. The mathematical formulas are presented as well. For each measure, it is assumed that we are concerned with the relationship between revenues per child ( $R_i$ ) and property wealth per child ( $W_i$ ) and that equity is improved as the measures show less and less of a relationship or approach zero in value.

Value Judgment Questions	Measures								
	Range	Restricted Range	Federal Range Ratio	Relative Mean Deviation	Permissible Variance	Variance	Coefficient of Variation	Standard Deviation of Logarithms	Gini Coefficient
1. Are all children taken into account in the measure?	No	No	No	Yes	No	Yes	Yes	Yes	Yes
2. Does the measure always show an improvement when revenues are transferred from one child to another lower in the distribution, without reversing the ranking of the children? (Such a transfer is often referred to as 'mean preserving'.)	No	No	No	No	No	Yes	Yes	Almost Always*	Yes
3. Does the measure always change when the revenues of each child are increased by a constant proportional amount?	Yes	Yes	No	No	No	Yes	No	No	No
4. Does the measure always change when the revenues of each child are increased by a constant absolute amount?	No	No	Yes	Yes	Yes	No	Yes	Yes	Yes
5. Does the measure record dollar changes at different levels of the distribution in the same way?	No	No	No	No	No	Yes	Yes	No	No
6. Is the mean used as a basis of comparison?	No	No	No	Yes	No	Yes	Yes	Yes	No
7. Is the median used as a basis of comparison?	No	No	No	No	Yes	No	No	No	No
8. Are all levels compared to one another as a basis for comparison?	No	No	No	No	No	No	No	No	Yes

\*Not always true for transfers that are made within the high end of the distribution.

Figure 1  
Value Judgments Inherent in Nine Dispersion Measures

1. The simple correlation (SIM CORR): The Pearson correlation coefficient between  $R_i$  and  $W_i$ .
2. The slope from the simple regression (SLOPE W): The slope coefficient which equals  $b_1$  from the regression  $R_i = a + b_1 W_i$ .
3. The slope from the quadratic regression (SLOPE W2): The slope coefficient which equals  $b_1 + 2b_2 \bar{W}$  from regression  $R_i = a + b_1 W_i + b_2 W_i^2$  where  $\bar{W}$  is the mean of  $W$ .
4. The slope from the cubic regression (SLOPE W3): The slope coefficient which equals  $b_1 + 2b_2 \bar{W} + 3b_3 \bar{W}^2$  from the regression  $R_i = a + b_1 W_i + b_2 W_i^2 + b_3 W_i^3$ .
5. The elasticity from the simple regression (ELAST W): The elasticity from the regression of  $R_i$  on  $W_i$  which equals (SLOPE W)  $\times \left( \frac{\bar{W}}{\bar{R}} \right)$ .
6. The elasticity from the quadratic regression (ELAST W2): The elasticity from the regression of  $R_i$  on  $W_i$  and  $W_i^2$  which equals (SLOPE W2)  $\times \left( \frac{\bar{W}}{\bar{R}} \right)$ .
7. The elasticity from the cubic regression (ELAST W3): The elasticity from the regression of  $R_i$  and  $W_i$ ,  $W_i^2$  and  $W_i^3$  which equals (SLOPE W3)  $\times \left( \frac{\bar{W}}{\bar{R}} \right)$ .

Each of the relationship measures contains a different set of value judgments. These value judgments are presented in Figure 2 in the form of six questions listed along the left side of the matrix. The seven measures are displayed along the top of the matrix, and the answers to each question are shown. The questions parallel the ones for the univariate dispersion measures, but they are reformulated in a bivariate or multivariate context. To illustrate the meaning of these value judgments, questions 2 and 5 have been selected for additional discussion.

Question 2 asks if the summary measure always shows an improvement when revenues are transferred from one child to another with lower revenues per child and lower property wealth per child. The transfer does not change the relative positions of the two children and retains the mean level of revenues per child and property wealth per child. This kind of transfer would seem to improve equity from many people's point of view, yet only the bivariate slope and the bivariate elasticity always show such an improvement. The other five measures may fail to register a positive change.

Question 5 asks if the measure changes (shows more or less equity) when property wealth per child increases or decreases by a given

Value Judgment Questions	Measures						
	Simple Correlation (SIM CORR)	Bivariate Slope (SLOPE W)	Quadratic Slope (SLOPE W2)	Cubic Slope (SLOPE W3)	Bivariate Elasticity (ELAST W)	Quadratic Elasticity (ELAST W2)	Cubic Elasticity (ELAST W3)
1. Are all children taken into account in the measure?	Yes	Yes	Yes	Yes	Yes	Yes	Yes
2. Does the measure always show an improvement when revenues (dependent variable) are transferred from one child to another with lower revenues per child and lower wealth per child without reversing the ranking of the children? (Such a transfer is often referred to as "mean preserving.")	Not Necessarily	Yes	Not Necessarily	Not Necessarily	Yes	Not Necessarily	Not Necessarily
3. Is the measure sensitive to equal percentage changes in revenues per child (dependent variable)?	No	Yes	Yes	Yes	No	No	No
4. Is the measure sensitive to equal additions to revenues per child (dependent variable)?	No	No	No	No	Yes	Yes	Yes
5. Is the measure sensitive to equal percentage increases in wealth per child (independent variable)?	No	Yes	Yes	Yes	No	No	No
6. Is the measure sensitive to equal additions to wealth per child (independent variable)?	No	No	No	No	Yes	Yes	Yes

Figure 2  
Value Judgments Inherent in Seven Relationship Measures

proportion for all children. The answer to this question is important when equity across time in a particular state is evaluated. A state may change the assessment equalization ratios for property values in all districts over time, thereby increasing or decreasing property values by constant percentages. Such administrative changes by themselves have nothing to do with equity, and the summary measure of equity should not change as a result of such administrative changes. Yet all three slope measures would show such a change. The other four measures (the simple correlation and the three elasticities) would not change in response to equal proportional additions or subtractions to property wealth per child.

The development of an equity standard for school finance reform involves a series of decisions about responses to the four questions: who, what, how, and how much. The responses can be interpreted as statements of value judgments. Any study of equity implicitly makes such judgments; we have tried in this section to make the alternatives explicit. In the next section, we demonstrate empirically that the use of different equity standards can influence the conclusions about the progress toward equity of a state's school finance system over time. The empirical work reinforces the theoretical importance of soliciting discussion within the education community on choices of standards.

## **EVALUATION OF SCHOOL FINANCE OVER TIME: EMPIRICAL EXAMPLES**

While alternative equity standards that incorporate different value judgments can be developed, the question remains: do the alternative standards yield different results? In this section, data from 21 states are utilized to document the contradictions among the equity standards when used to assess the equity of a particular state's school finance system over time.<sup>7</sup> We do not evaluate the contradictions among all possible equity standards; we only focus on one group (children) and one object (revenues). First, the contradictions among the nine children-revenue-disparity standards are examined and, second, the contradictions among the seven children-revenue-relationship standards are described, where wealth per pupil is the independent variable. These two analyses follow a brief discussion of the data and methodology.<sup>8</sup>

The data base has been constructed by a group of researchers and policy analysts in universities, research organizations, and the federal government who are known as the School Finance Cooperative.<sup>8</sup> The Cooperative is simultaneously exploring the methodology of equity measurement and accumulating a data base of equity measures in about

twenty states. Various members of the Cooperative have analyzed the school finance systems in different states, and since a national data base for all districts in all states does not yet exist, the Cooperative's pooled data base is unique.

Certain definitional assumptions were built into the data set, but the structure of the data in individual states prevented complete comparability. However, since we are examining the behavior of various standards over time within a state, and since the data are always comparable for each state intertemporally, comparability problems should not affect conclusions about the standards.

The equity standards examined are based on children as the group, and the pupil count utilized is the average daily membership for each district. The object is dollars per pupil in each district, where the dollars include all local and state revenues except revenues for debt service and capital. State equalized assessed value per pupil in each district is the wealth variable used in the relationship measures. While almost all states use some form of statewide equalization, very few states equalize to the full market value. Although the original data consist of observations on number of pupils, per pupil revenues, and per pupil wealth for each district in a state, the equity standards are computed using the pupil as the unit of analysis. Each pupil in a district is assigned the district level per pupil revenue and per pupil wealth, and the equity standards are computed using this distribution of pupils. This procedure can also be viewed statistically as weighting the observations in each district by the number of pupils in the district. The pupil unit of analysis thus allows larger districts to exert a greater influence on the distribution of per pupil revenues in the state. An alternative computational procedure would be to count each district equally, thereby forming a distribution of per pupil observations for each district rather than for each pupil. The results for this alternative procedure, known as the district unit of analysis, are reported in footnotes.

The nine dispersion measures and the seven relationship measures discussed earlier and listed in Figures 1 and 2, respectively, are employed in the following analyses. For the dispersion and relationship measures, data are available for twenty-one and twenty states, respectively, for various years. The analyses examine the change in a state from the earliest to the latest year available, which varies state by state, rather than comparisons among all years. Furthermore, in states with multiple district types, only the unified K-12 districts are included. If all available years and all district types were included, a considerably larger data set could be generated. However, the results from the analysis of the standards using this larger data set are virtually identical to the results reported here.<sup>9</sup> While the results for the degree of contradiction among

the measures may be unaffected by the selection of a data base for analysis, the conclusions regarding movement toward or away from equity in a particular state may be sensitive to the year chosen.

It should be clear from this description of the data and computational procedures that a particular set of assumptions has been built into the analyses, and for that reason attention should be focused on the agreement and contradictions among the standards and not on the results for a particular state. For example, the use of other objects, such as instructional expenditures, local plus state direct revenues, or total revenues including federal funds, or the use of other years may be preferred in an assessment of a particular state over time. Thus, the reader is warned against judging a particular state's progress toward or away from equity solely on the analyses presented here.

### Intertemporal Analysis of Dispersion Equity Standards

The question addressed in this part may be stated as follows:

*When a number of dispersion equity standards, computed using the pupil unit of analysis, are used to determine whether a state has become more or less equitable between two points in time, do the dispersion standards agree?*

The least restrictive way to assess the extent to which there is agreement or contradiction among the dispersion standards is to compute the percentage of the time all nine dispersion measures agree. An analysis of the behavior of the standards over time is displayed in Table 1, and this shows that complete agreement among all nine standards occurred in only three of the 21, or 14 percent of the cases (column 3).<sup>10</sup>

Since complete agreement is rare, it would be helpful to know whether there is more agreement among certain subsets of the measures, and it is logical to formulate the subsets based on the value judgments described earlier. An important value judgment is whether the standard is sensitive to equal percentage changes. Only the range, restricted range, and variance are sensitive to equal percentage changes, and Table 1 shows that the remaining six measures that are not sensitive to equal percentage changes agree in six cases (column 4), in addition to the three cases of total agreement. Therefore, in nine of the 21 cases, or 43 percent of the time, there is agreement among the six dispersion standards that are not sensitive to equal percentage changes (columns 3 and 4).

A second important value judgment is whether the standard is sensitive to mean preserving transfers. Only the Gini coefficient and coefficient of variation are *both* insensitive to equal percentage changes and sensitive to mean preserving transfers. However, since the standard deviation of

logarithms is insensitive to only a very small number of mean preserving transfers, we include the standard deviation of logarithms along with the Gini coefficient and the coefficient of variation in the analysis. Table 1 indicates those three standards agree when all the standards agree (three cases), when the group of six standards agree (six cases), and in nine additional cases (column 5). As a result, in eighteen of the 21, or 86 percent of the cases, these three standards agree (columns 3, 4, and 5).<sup>11</sup>

**Table 1**  
**Agreement and Contradictions Among Dispersion Measures**  
**in States Over Time, Pupil Unit Analysis\***

(1) State	(2) Years	(3) All Agree	(4)** All Agree Except One or More of R, RR, VAR	(5)*** COV, LOG, GINI Agree; Contra- diction Among One or More of FRR, RMD, PV, and Possibly R, RR, VAR as Well	(6) Contradiction Among COV, LOGS, GINI, and Possibly Others as Well	(7) Conclu- sions from COV, LOGS, GINI
Ala.	72/73- 75/76	—————→		R, RR, VAR, PV		More Equitable
Cal. Unified	70/71- 74/75	—————→	VAR			More Equitable
Colo.	72/73- 74/75	—————→			All less except PV, LOGS	Uncertain
Fla.	72/73- 75/76	—————→		PV		Less Equitable
Ga.	72/73- 75/76	—————→			All less except FRR, RMD, GINI	Uncertain
Ill. Unified	72/73- 75/76	————→X				Less Equitable
Kan.	72/73- 74/75	—————→			All less except PV, LOGS	Uncertain
Ky.	72/73- 75/76	—————→		PV		Less Equitable
La.	72/73- 75/76	————→X				Less Equitable
Mass.	72/73- 75/76	—————→	RR, VAR			More Equitable
Mich. Unified	71/72- 74/75	—————→		R, RR, VAR, PV		More Equitable
Minn.	71/72- 75/76	—————→	RR, VAR			More Equitable
Miss.	71/72- 75/76	—————→		R, RR, VAR, FRR		More Equitable
Mo. Unified	74/75- 75/76	—————→	R, VAR			More Equitable
N.J.	74/75- 77/78	—————→		R, RR, VAR, FRR		More Equitable

Table 1 (cont.)

## Agreement and Contradictions Among Dispersion Measures in States Over Time, Pupil Unit Analysis\*

(1) State	(2) Years	(3) All Agree	(4)** All Agree Except One or More of R, RR, VAR	(5)*** COV, LOG, GINI Agree; Contra- diction Among One or More of FRR, RMD, PV, and Possibly R, RR, VAR as Well	(6) Contradiction Among COV, LOGS, GINI, and Possibly Others as Well	(7) Conclu- sions from COV, LOGS, GINI
N. M.	72/73- 75/76	—————→		R, RR, VAR, PV		More Equitable
N. C.	72/73- 75/76	————→	R, RR, VAR			More Equitable
S. C.	72/73- 75/76	→X				Less Equitable
S. D.	73/74- 75/76	—————→		RR, VAR, FRR		More Equitable
Tex.	74/75- 75/76	————→	R, RR, VAR			More Equitable
Wash.	70/71- 74/75	—————→		RMD		Less Equitable
All States		3/21 (14%)	6/21 (29%)	9/21 (43%)	3/21 (14%)	

\*Columns 3, 4, 5, and 6 are mutually exclusive. Each state has an entry in only one of them. The symbols have the following meanings:

- R = Range
- RR = Restricted Range
- FRR = Federal Range Ratio
- RMD = Relative Mean Deviation
- PV = Permissible Variance
- VAR = Variance
- COV = Coefficient of Variation
- LOGS = Standard Deviation of Logarithms
- GINI = Gini Coefficient

\*\*Entries in this column indicate measures that contradict with the six equity measures: FRR, RMD, PV, COV, LOGS, GINI.

\*\*\*Entries in this column indicate measures that contradict with the three equity measures: COV, LOGS, GINI.

Thus, there are many contradictions among all nine dispersion standards when used intertemporally. Only if particular value judgments are selected, thereby reducing the number of standards to as few as three, will agreement be close to 90 percent. However, it may be impossible in a policy evaluation to assume such an agreement on value judgments, so the use of multiple standards that reflect differing value judgments may be in order. For example, if there is disagreement over whether there is concern either with those pupils or districts below the median revenue level or with all pupils or districts, then the use of the permissible variance in combination with the Gini coefficient, coefficient of

variation, and/or standard deviation of logarithms may be called for. Although this is just an example, the level of contradictions indicates that there may be many situations where multiple measures should be employed.

### Intertemporal Analysis of Relationship Equity Standards

The question addressed in this part may be stated as follows:

*When a number of relationship equity standards based on wealth, computed using the pupil unit of analysis, are used to determine whether a state has become more or less equitable over time, do the relationship standards agree?*

For the seven bivariate or multivariate relationship standards described earlier, when applied to the twenty-state sample, all show movement in the same direction in eight of the twenty, or 40 percent of the cases, as shown in Table 2 (column 3).<sup>12</sup> Although this level of agreement is higher than was observed for the dispersion measures, it is still far from complete agreement.

**Table 2**  
Agreement and Contradictions Among Relationship Measures in States Over Time, Pupil Unit of Analysis\*

(1) State	(2) Years	(3) All Agree	(4)* All Elasticity Measures Agree but Contradiction Among One or More of SIM CORR, SLOPE W, SLOPE W2, SLOPE W3	(5) Contradiction Among Elasticity Measures and Perhaps Others as Well	(6) Conclusion from Elasticity Measures
Cal. Unified	70/71-74/75	→X			More Equitable
Col.	72/73-74/75			All less except SIM CORR, ELAST W	Uncertain
Fla.	72/73-75/76		SLOPE W, SLOPE W2, SLOPE W3		Less Equitable
Ga.*	72/73-75/76			All less except SLOPE W2, ELAST W2	Uncertain
Ill. Unified	72/73-75/76			All less except SIM CORR, ELAST W	Uncertain
Kan.*	72/73-74/75	→X			Less Equitable
Ky.	72/73-75/76	→X			Less Equitable
La.	72/73-75/76			All more except SLOPE W, SLOPE W2, SLOPE W3, ELAST W3	Uncertain

Table 2 (cont.)

Agreement and Contradictions Among Relationship Measures in States Over Time, Pupil Unit of Analysis\*

(1) State	(2) Years	(3) All Agree	(4)** All Elasticity Measures Agree but Contradiction Among One or More of SIM CORR, SLOPE W, SLOPE W2, SLOPE W3	(5) Contradiction Among Elasticity Measures and Perhaps Others as Well	(6) Conclu- sion from Elasticity Measures	
Mass.*	72/73- 75/76	→		More SIM CORR SLOPE W ELAST W	Less SLOPE W2 ELAST W2	Uncertain
Mich.	71/72- 74/75	→X				More Equitable
Minn.	71/72- 75/76	→		All less except SIM CORR, SLOPE W, ELAST W		Uncertain
Miss.	71/72- 75/76	→		All less except SLOPE W2, ELAST W2		Uncertain
Mo. Unified	74/75- 75/76	→X				More Equitable
N. J.	74/75- 77/78	→X				Less Equitable
N.M.	72/73- 75/76	→		All more except SLOPE W3, ELAST W3		Uncertain
N. C.	72/73- 75/76	→X				More Equitable
S. C.	72/75- 75/76	→	SLOPE W, SLOPE W2, SLOPE W3			More Equitable
S. D.	73/74- 75/76	→X				More Equitable
Tex.	74/75- 75/76	→	SIM CORR, SLOPE W, SLOPE W2, SLOPE W3			More Equitable
Wash.	70/71- 74/75	→	SIM CORR			Less Equitable
All States		8/20 (40%)	4/20 (20%)		8/20 (40%)	

\*Columns 3, 4, and 5 are mutually exclusive. Each state has an entry in only one of them. The symbols have the following meanings:

SIM CORR = Simple Correlation  
 SLOPE W = Bivariate Slope  
 SLOPE W2 = Quadratic Slope  
 SLOPE W3 = Cubic Slope  
 ELAST W = Bivariate Elasticity  
 ELAST W2 = Quadratic Elasticity  
 ELAST W3 = Cubic Elasticity

\*\*Entries in this column indicate measures that contradict with the three elasticity measures: ELAST W, ELAST W2, ELAST W3.

\* SLOPE W3 and ELAST W3 not computed.

We can analyze the behavior of the relationship standards further by determining whether the levels of agreement are altered when we examine a subset of the standards grouped according to the value judgments identified earlier. Since the three elasticity measures incorporate similar (but not identical) value judgments and since the elasticity measures are not sensitive to changes in state equalization assessment ratios, the behavior of the three elasticity standards as a group is analyzed. Table 2 shows, however, that there is agreement among the three elasticity standards in only twelve out of the twenty, or 60 percent of the cases (columns 3 and 4).<sup>13</sup>

This evaluation of a particular group of relationship standards from twenty states indicates that the selection of a standard can critically influence the conclusions drawn from an intertemporal analysis of equity. If there is not agreement on the value judgments, then these results emphasize the need to select standards that incorporate a wide range of value judgments and probably argue for the use of multiple measures.

## CONCLUSIONS

Our basic point can be stated rather simply: the assessment of equity in school finance involves the imposition of certain value judgments, and, furthermore, these value judgments can be evaluated analytically and empirically. Some may argue that value judgments, such as the ones discussed here, are best left behind the scenes, but we disagree. The choice of value judgments influences the outcome of the evaluation, and the value judgments are too complex to allow an implicit resolution.

Our analysis of the behavior of the children-dollar-equity standards over time in twenty states shows that there is a fair degree of contradiction among the standards. A reasonable hypothesis might be that if a state "reformed" its school finance system during the period of study, then the standards would be in agreement, but if a state kept its finance system pretty much the same, the standards would show more conflict. This hypothesis is not borne out by the data. Contradictions among the standards occur for states with and without reform, and, likewise, both reform and nonreform states show agreement among the standards.

While our conclusions may be straightforward, they raise some rather perplexing questions, such as the following:

- What are the procedures, in a public policy context, by which the value judgments to be included in the equity standards are identified and agreed on?

- When multiple standards conflict, how can the trade-offs among the standards be determined?
- If standards are identified as desirable, what policy instruments are available to move a system toward these standards?

That these questions are important yet difficult to answer emphasizes the fact that much concerning the equity of public education is yet to be learned. But what we do know should become part of equity evaluations.

## NOTES

1. The research on which this paper is based was carried out at the Public Policy Research Institute, Graduate School of Public Administration, New York University, New York, N. Y. Funds were provided by the Ford Foundation and by the Education Commission of the States through a grant from The National Institute of Education. We would like to thank Chris Hakusa for excellent computer programming assistance and Karen Gruhn for fine secretarial assistance.
2. See the *Journal of Education Finance* 3 (Spring 1978): 375-535.
3. Thomas Jones in a recent issue of this journal has discussed the taxpayer's point of view and commented on the children's point of view. See T. Jones, "The School Property Tax Reconsidered," *Educational Administration Quarterly* 14 (Spring 1978): 98-114.
4. Other relationship-type measures, such as correlations, slopes, or elasticities, could be used to measure vertical equity. These other measures are described in the next section.
5. The *Serrano* cases in California (1971, 1977) are the most famous examples. The *Levittown* decision in New York (1978) is one of the most recent examples.
6. See R. Berne, "Alternative Equity and Equality Measures: Does the Measure Make a Difference?" in *Selected Papers in School Finance, 1978*, ed. E. O. Tron (Washington, D.C.: U.S. Office of Education, 1978); A. Sen, *On Economic Inequality* (New York: Norton, 1973); A. Atkinson, "On the Measurement of Inequality," *Journal of Economic Theory* 2 (1970): 244-63.
7. For an analysis of the contradictions among the standards when used to rank a set of states at one point in time see R. Berne and L. Stiefel, "A Methodological Assessment of Educational Equality and Wealth Neutrality Measures," Public Policy Research Institute, Graduate School of Public Administration, New York University, and Papers in Education Finance, paper no. 17, Education Commission of the States, Denver, Colo., July 1978.
8. The following School Finance Cooperative members contributed data: Education Commission of the States, Education Policy Research Institute

Intercultural Development Research Institute (in cooperation with Professor Walter Garms, University of Rochester), Center for Study of Educational Finance at Illinois State University, Lawyers Committee for Civil Rights Under Law, National Conference of State Legislatures, and Rand Corporation.

9. See Berne and Stiefel, "A Methodological Assessment of Education Equality and Wealth Neutrality Measures."
10. For the district unit of analysis, complete agreement occurred in 29 percent of the cases.
11. For the district unit of analysis, the corresponding figure is 90 percent.
12. For the district unit of analysis, complete agreement among all seven measures occurred for 40 percent of the cases also.
13. For the district unit of analysis, the three elasticity measures agree in 60 percent of the cases also.