Poverty, economic growth, and average exit time

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Abstract

A simple transformation of the Watts poverty index yields a “meaningful” measure with appealing ordinal properties and a natural interpretation in terms of the potential for economic growth to alleviate poverty. The index is illustrated with data from Bangladesh and Bolivia. © 1998 Elsevier Science S.A.

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1. Introduction

Economists have proposed a wide variety of indices of poverty, each with competing attributes and justifications. For the most part, though, only two indices are used regularly in poverty analysis: the headcount index and the poverty gap. The former simply measures the number of individuals below an income threshold, and the latter measures the total size of transfers needed to bring poor individuals up to the threshold. Their popularity remains strong despite cogent critiques: in particular, neither measure reflects changes in the distribution of income below the poverty line (Sen, 1976).

In 1968, Harold Watts proposed a simple alternative measure that is distributionally-sensitive and decomposable as the weighted sum of poverty measures of population subgroups.¹ The index has appealing ordinal properties, and below it is shown that a simple linear transformation of the index gives it cardinal properties that can be useful as well. This modified Watts measure is termed the “average exit time.”

The original Watts measure is defined as

\[ W = \frac{1}{N} \sum_{i=1}^{q} [\ln(z) - \ln(y_i)], \]

where there are \( i \) individuals in the population indexed from 1 to \( N \) in ascending order of income and \( q \) is the number of people with income \( y_i \) below the poverty line \( z \).

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¹The attributes of the Watts index and a characterization are described by Zheng (1993).

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The average exit time, $T_g$, is simply $W/g$, where $g > 0$ is a hypothetical growth rate of incomes of the poor population. The index reflects the average number of years that it would take the population to exit poverty if it were possible to ensure that all incomes grow at rate $g$.

The average exit time is derived below and applied to data from urban Bolivia and rural Bangladesh in Section 4.

2. Exit time

A household’s expected exit time is defined as the time it will take it to reach a given poverty line via income growth. If the income of household $i$ grows at a constant positive rate $g$ per year, the relationship of the poverty line to current income can be written

$$ z = y_i (1 + g)^t_i. $$

Taking the logarithm and solving for $t_i$ yields

$$ t_i = \frac{\ln(z) - \ln(y_i)}{g}, \quad (1) $$

for households below the poverty line. For households above the poverty line, $t_i = 0$. The approximation $\ln(1 + g) \approx g$ is used here. Eq. (1) holds exactly for exponential growth such that $z = y_i \exp(t_i g)$.

In Bangladesh, for example, the median poor rural household in the 1988–89 Household Expenditure Survey spent Tk. 284 per month per capita relative to the poverty line of Tk. 370 (in 1989, Tk. 32.1 = $1). So, if their expenditures grew at 3% per year, it would take 8.8 years to reduce half of rural poverty through growth alone. While rough, this sort of calculation gives a sense of the contribution that growth can make to poverty alleviation. Kanbur (1987), for example, suggests using the exit time for the average poor person as a simple poverty measure:

$$ t_{i_{\text{avg}}} = \frac{\ln(z) - \ln(\mu_p)}{g}, \quad (2) $$

where $\mu_p$ is the average income of poor households. The measure has appeal, but it is not sensitive to the distribution of income below the poverty line since, like the poverty gap, it is based on the average income of the poor.

The average exit time, in contrast, reflects differences between extreme and moderate poverty. The average exit time is simply $t_i$ averaged over the entire population:

$$ T_g = \frac{1}{N} \sum_{i=1}^{N} t_i = \frac{1}{N} \sum_{i=1}^{q} \frac{\ln(z) - \ln(y_i)}{g} = W/g. \quad (3) $$

Calculation of the index is simple, involving only an ordinal transform of the Watts measure.
3. Properties and relationships to other measures

The measure nests the Theil-L index of inequality. The Theil index is calculated as

$$L_p = \frac{1}{q} \sum_{i=1}^{q} [\ln(\mu_p) - \ln(y_i)].$$

when applied just to the $q$ households below the poverty line. The Theil index is bounded between 0 and 1, but by dividing $L_p$ by $g$, the index can be put in units of time, yielding $L_g = L_p/g$. Substituting Eqs. (2) and (4) into (3) yields an equivalent form for the average exit time:

$$T_g = H(\bar{t}^{avg} + L_g).$$

When all incomes below the poverty line are equally distributed, $L_g = 0$ and the average exit time is simply $H\bar{t}^{avg}$. Thus, the distributional-sensitivity comes directly from the nesting of the Theil index in much the same way as the Sen (1976) index nests the Gini coefficient.

Because the measure is built around a simple transformation of the Theil index, it is easily checked that the measure satisfies the monotonicity and transfer axioms. The monotonicity axiom states that a reduction in the income of a poor household must increase the poverty measure, holding all else constant. The transfer axiom states that a positive transfer from one poor household to any less poor household must increase the poverty measure.

As a result of its additively separable form, the measure is also decomposable into the population-weighted measures of subpopulations of the poor, a feature that has proven useful for poverty analysis (Foster et al., 1984). Thus the Watts index and average exit time have nearly identical properties to the Foster–Greer–Thorbecke measures employed widely in World Bank poverty assessments (Lipton and Ravallion, 1995).

4. Application to rural Bangladesh and urban Bolivia

The concept of exit time is illustrated below for a hypothetical growth rate of 3%. The data come from three rounds of the Encuesta Integrada de Hogares of Urban Bolivia (1989–92) and the rural sample from the 1988–89 Household Expenditure Survey of Bangladesh. These are large, representative surveys, covering two low-income economies where poverty alleviation has been a continuing policy goal.

\[^{3}\text{When income below the poverty line is distributed equally, the Sen index reduces to } H \text{ multiplied by the poverty gap measure. This is the analogue to } H\bar{t}^{avg} \text{ here. As with the Watts measure and the Theil index, the average exit time is undefined when any household has zero income since } t_i \to \infty \text{ as } y_i \to 0. \text{ Zero income is most likely due to under-reporting and the problem should be addressed in the data collection process or by adding small amounts of income to zero values.}\]

\[^{4}\text{The Watts index has potential appeal relative to the commonly-used squared poverty gap of Foster–Greer–Thorbecke measure by being “transfer sensitive”: regressive transfers between very poor households increase measured poverty more than regressive transfers between less poor households. A potential disadvantage is that weighting is not smooth at the poverty line (Lipton and Ravallion, 1995). Neither feature is likely to affect poverty rankings much in practice, relative to other properties.}\]

\[^{5}\text{In 1990, 3.2 Bs. } = \$1. \text{ In 1989, Tk. 32.1 } = \$1 \text{ and 2.7 Bs. } = \$1.\]
While the headcount index remained fairly constant at 55%, the average exit time in urban Bolivia increased from 10.9 to 13.2 years during the period. The 20% increase reflects a decline in the average income of poor households from 100.09 Bs. to 90.39 Bs., pushing the poverty gap up by 14%. This was a period of worsening distribution: the income decline occurred while average incomes of the broader population grew by nearly 4%.\(^5\) At the same time, income inequality of poor households, as measured by the Theil index, increased by over 20%.

The average exit time provides a way to put these pieces together to tell a coherent story about the changes. First, since the income decline of the poor averaged just under 3% per year, by construction the exit time of the average poor household, \(t_{avg}^{3\%}\), should gain just under a year for each of the 3.7 years between rounds 1 and 5 of the survey. Table 1 shows this exactly: \(t_{avg}^{3\%}\) increased by 3.4 years between March 1989 and October 1992. As shown in Eq. (5), \(T_{3\%}/H\) is the sum of \(t_{avg}^{3\%}\) plus the rescaled Theil-L inequality index. Inequality increases the average exit time of the poor population, \(T_{3\%}/H\), by nearly 20% over \(t_{avg}^{3\%}\), but the change in inequality over the period contributes only an extra 0.7 years to the change in \(T_{3\%}/H\). Thus, once measured inequality is put into comparable terms, it can be seen directly that its contribution to the change in poverty is minor.

These effects are summarized by the average exit time, \(T_{3\%}\). The increase from 10.9 to 13.2 years combines these forces and testifies to the continuing need for economic growth which reaches down to the poorest households in urban Bolivia.

The situation in rural Bangladesh shows the potential for growth there as well. The poverty line in rural Bangladesh is much lower than in urban Bolivia, $11 versus $63 in 1989, and this explains the low average exit time of 4.9 years. The low average exit time also reflects a low degree of inequality among poor households. The calculations suggest that a decade of steady growth at 3% can eliminate much of measured poverty. But it would take at least three generations of steady growth at 3% for the average poor household in Bangladesh to reach Bolivia’s poverty line.

Table 1
Poverty measures and average exit time (assuming 3% average growth per year). Urban Bolivia and rural Bangladesh

<table>
<thead>
<tr>
<th></th>
<th>Encuesta Integrada de Hogares, urban Bolivia</th>
<th>Rural Bangladesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headcount index (H)</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td>Average exit time ((T_{3%}))</td>
<td>10.9 years</td>
<td>12.0 years</td>
</tr>
<tr>
<td>Exit time of the avg.</td>
<td>17.5 years</td>
<td>18.5 years</td>
</tr>
<tr>
<td>Poor household ((t_{avg}^{3%}))</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>Inequality of poor (Theil-L + 3%)</td>
<td>69.1 Bs.</td>
<td>72 Bs.</td>
</tr>
<tr>
<td>Poverty gap</td>
<td>169.19 Bs.</td>
<td>169.19 Bs.</td>
</tr>
<tr>
<td>Poverty line</td>
<td>3874</td>
<td>6353</td>
</tr>
</tbody>
</table>

Notes: Expenditure data is per capita per month, calculated using population weights. Bolivian data is in September 1990 Bolivianos. Bangladesh data is in 1989 takas.

\(^5\)Since economic growth pushes some households across the poverty line, it is possible that all households have positive growth rates but, at the same time, the average income of poor households may fall—as the least poor households exit poverty, poorer households are left behind, lowering the average income of those remaining below the poverty line.
5. Meaningful poverty measurement

The headcount and the poverty gap are set apart from other poverty measures by being “meaningful”—i.e., there is particular interest in the cardinal representations of the indices themselves (Foster, 1994). Squaring, cubing, or positively transforming these indices in any other way makes them much less informative measures. Their popularity suggests that for many purposes this cardinal meaningfulness outweighs other disadvantages of the two measures.

Like the poverty gap, the average exit time is meaningful through describing an interesting “if–then” relationship. The poverty gap indicates if there is perfectly targeted, costless redistribution then how much money will be required to completely eliminate poverty. These assumptions are clearly unrealistic—redistribution will never be perfectly targeted nor costless—but the poverty gap nevertheless conveys useful information.

Similarly, the average exit time is also based on “best case” assumptions. Even though growth is unlikely to be constant or uniform across households, the average exit time provides a simple metric of the potential for poverty alleviation through growth. By using “best case” assumptions rather than context-specific assumptions, the average exit time is simple and comparable across regions and time periods.

Achieving broad-based economic growth is firmly on policy agendas, and the accumulating evidence shows that achieving growth and reducing poverty can be strongly complementary (e.g., Deininger and Squire, 1996). Developing analytical tools to consider poverty and growth jointly is thus important.

The usefulness of this particular measure depends on the relevance of the hypothetical question. While the index is built around the dimension of time, it remains a measure of static conditions. It does not measure the actual duration of poverty nor average expected duration based on individual-specific factors. Raising the hypothetical growth rate $g$ here trivially reduces the average exit time, so comparisons across measures with different hypothetical growth rates shed little light.

The value of the index is in mapping an income distribution into a simple metric that conveys information about the nature of poverty. If an individual earns $1000 per year, it would take them 22 years to reach a poverty line of $3000 if their income were to grow at a real rate of 5% annually. It would take just 8 years for someone starting at $2000 per year. These differences in exit time reflect a potentially important dimension by which to gauge what it means to have an annual income of $1000 versus $2000. The average exit time presents one simple way to begin incorporating these dimensions into poverty analyses.

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