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## **Conversions between percents, decimals, and fractions**

Given the frequency that percents occur, it is very important that you understand what a percent is and are able to use percents in performing calculations. This unit will help you learn the skills you need in order to work with percents.

Percent means "per one hundred" Symbol for percent is %

For example, suppose there are 100 students enrolled in a lecture class. If only 63 attend the lecture, then 63 per 100 students showed up. This means 63% of the students attended the lecture.

If we take a closer look at this, we can see that the fraction of the students that attended the lecture is  $63/100$ , and this fraction can be expressed as 0.63.

- 63% is the same as  $63/100$
- 63% is the same as 0.63

This example shows how decimals, fractions, and percents are closely related and can be used to represent the same information.

### ***Converting between Percents and Decimals***

#### **Convert from percent to decimal:**

To change a percent to a decimal we divide by 100. This is the same as moving the decimal point two places to the left. For example, 15% is equivalent to the decimal 0.15.

$$15\% = \frac{15}{100} = .15$$

Notice that dividing by 100 moves the decimal point two places to the left.

15. gives us .15

### Example

What decimal is equivalent to 117%?

**NOTE:** Since 117% is greater than 100%, the decimal will be greater than one.

To convert 117% to a decimal, we divide by 100. When we do this, we get the decimal 1.17.

$$117\% = \frac{117}{100} = 1.17$$

### Convert from decimal to percent:

To change a decimal to a percent, we multiply by 100. This is the same as moving the decimal point two places to the right. To change the decimal 0.22 to a percent, we move the decimal point two places to the right and get 22%.

$$.22 \times 100 \text{ is the same as } 22 \quad .22 \times 100 = 22\%$$

### Example

Convert 0.55 to a percent.

To convert 0.55 to a decimal, we multiply by 100. When we do this we get 55%.

$$.55 \times 100 = 55\%$$

**NOTE:** The decimal .555 is equivalent to 55.5%.

### *Converting between Percents and Fractions*

To convert a percent to a decimal, we divide by 100. This is the same thing we do to convert a percent to a fraction. The number before the (%) sign becomes the

numerator, and the denominator is 100. Once this is done, we can simplify the fraction. For example, if we wish to convert 10% to a fraction, we divide the 10 by 100 and simplify the fraction.

$$10\% = \frac{10}{100} = \frac{1}{10}$$

### Example

Convert 0.5% to a fraction.

1. Divide the percent by 100 to get a fraction.

$$0.5\% = \frac{0.5}{100} = \frac{5}{1000}$$

2. Simplify the fraction.

$$\frac{5}{1000} = \frac{1}{200}$$

**NOTE:** .5% = .005 = 5/1000 = 1/200

### Convert from Fraction to Percent

Now let's look at converting a fraction to a percent.

1. Convert the fraction to a decimal.
2. Convert the decimal to a percent.

If we want to convert 1/4 to a percent, we divide 1 by 4 to get the decimal. Then we convert that decimal to a percent.

1. Convert the fraction to a decimal.

$$\frac{1}{4} = 4 \overline{)1.00} \begin{array}{r} .25 \\ \end{array}$$

2. Convert the decimal to a percent.

$$0.25 \times 100 = 25\%$$

### **Example**

Convert the fraction  $\frac{3}{5}$  to a percent.

1. Convert the fraction to a decimal.

$$\frac{3}{5} = .6$$

2. Convert the decimal to a percent. (Multiply the decimal by 100.)

$$0.6 \times 100 = 60\%$$

**NOTE:** Converting fractions to percents can also be done using proportions. This method will be discussed in the section on proportions.

## **Operations with percents**

### ***Addition and Subtraction of Percents***

These operations can be done on percents without any conversion needed.

### **Example**

When you add 10% and 20% you get 30%.

$$10\% + 20\% = 30\%$$

When you subtract 10% from 20% you get 10%.

$$20\% - 10\% = 10\%$$

### ***Multiplication and Division of Percents***

With multiplication and division, percents should be converted to either decimals or fractions before performing these operations. This must be done because of how the decimal point is carried during these operations.

When performing any multiplication or division with percents, you need to go through the following steps:

1. Convert any percents to decimals (or a fraction).
2. Perform the operation specified.

3. Convert the result back to a percent.

### **Example of Multiplying Percents**

When we multiply 50% by 25% (this would be 25% of 50%), we must:

1. Convert any percents to decimals (or a fraction).

$$\begin{aligned}50\%/100 &= 0.5 \\25\%/100 &= 0.25\end{aligned}$$

2. Perform the operation specified.

$$0.5 * 0.25 = 0.125$$

3. Convert the result back to a percent.

$$0.125 * 100 = 12.5\%$$

### **Example of Dividing Percents**

When we divide 25% by 75% we must follow the steps outlined above.

1. Convert any percents to decimals (or a fraction).

$$\begin{aligned}25\%/100 &= 0.25 \\75\%/100 &= 0.75\end{aligned}$$

2. Perform the operation specified.

$$0.25 / 0.75 = 0.3333$$

3. Convert the result back to a percent.

$$0.3333 * 100 = 33.33\%$$

### **Percentage of a number**

To find a percentage of a number, write the percentage as a decimal or fraction. Then multiply by the given number.

#### **Example**

1. Find 56.4% of 125.

Write the percentage as a **decimal** and multiply by 125.

$$56.4\% = 0.564$$

$$\begin{aligned} 56.4\% \text{ of } 125 &= 0.564 \times 125 \\ &= 70.5 \end{aligned}$$

70.5 is 56.4% of 125.

2. Find 75% of 84.

Write the percentage as a **fraction** and multiply by 84.

$$\begin{aligned} 75\% &= 3/4 \\ 75\% \text{ of } 84 &= 3/4 \times 84 \\ &= 3 \times 21 \\ &= 63 \end{aligned}$$

63 is 75% of 84.

### ***Percent of a Total***

Another common application of percent is to measure one category as a percent of another category. This is like asking, "650,000 is what percent of 1,000,000?" For

### **Example**

Total budget = \$1 million

Personnel = \$650,000

Personnel as percent of total =  $(650,000/1,000,000) \times 100 = 65\%$

The personnel budget is 65% of the total budget.

### **Percent change**

One of the most common applications of percent is to measure change from one time period to another. Percent changes are widely used in economics, finance, and statistics.

Percent change is the change in the frequency of something over time. For example, if your revenue from property taxes went up from \$10 million to \$11 million, by instinct many of us would say that this was a 10 percent change in property taxes.

That is,

$$(\text{Change between two periods} / \text{base level}) \times 100 = \text{percent change}$$

The percent change in a quantity equals the change divided by the base.

Suppose a variable X has the initial value (or base)  $X_0$ , then changes to the value  $X_1$ . Then, the change in X =  $\Delta X = X_1 - X_0$

$$\text{Percent change} = \Delta X / X_0 = [(X_1 - X_0) / X_0] \times 100$$

$$\text{Equivalently, percent change} = [(value 2 - value 1) / value 1] \times 100$$

Note that the Greek letter delta  $\Delta$  is often used to mean “change in.”

One of the decisions that must be made is what base to use to calculate the percent change. The usual practice is to take the value for the first period, so that the percent change is interpreted as percent change from the previous period. (However, there are a number of applications where the average of the two periods might be used.)

$$\begin{aligned} \text{Percent change} &= [(value 2 - value 1) / value 1] \times 100 \\ &= [(11,000,000 - 10,000,000) / 10,000,000] \times 100 \\ &= 10 \% \end{aligned}$$

### Example

Suppose the variable X increases from 8 to 12. Change in X =  $\Delta X = \underline{\quad? \quad}$ . If 8 = initial value, % change in X = ?.

*Solution:* Suppose the variable X increases from 8 to 12. Change in X =  $\Delta X = 4$ . If 8 = initial value, % change in X = **0.50 = 50%**.

### Example

Suppose the variable Y decreases from 12 to 8. Change in Y =  $\Delta Y = \underline{\quad? \quad}$ . If 12 = initial value, % change in Y = ?.

*Solution:* Suppose the variable Y decreases from 12 to 8. Change in Y =  $\Delta Y = 4$ . If 12 = initial value, % change in Y = **-0.333 = -33.3%**

Percent change is also sometimes referred to as the **Growth Rate**.

Consider the United States' Gross Domestic Product (GDP) during the Great Depression. The data (in billions of \$USD) are:

Year	U.S. GDP (in billions)
1929	\$103.8
1930	91.1
1931	76.4
1932	58.6
1933	56.2

Suppose we want to figure out the percentage change in GDP during the Depression. We can see that, from 1929 to 1930, US GDP fell by \$103.8B - \$91.1B = \$12.7B. But how big is this? From the first to the second quarter of 1997, U.S. GDP grew by \$108.4B (from \$7955B to \$8063B). The economy has gotten much larger in the intervening span of time. The percentage change in GDP tells us how large the fall in GDP was, relative to the total amount of \$103.8B available to be lost.

So we calculate the percentage change in GDP as  $(91.1 - 103.8)/103.8$ , which is -0.1223. To make this decimal into a percentage, we multiply by 100 (move the decimal point 2 spots to the right).

The result is a growth rate of -12.23% (or a fall of 12.23%) from 1929 to 1930.

Now, try to calculate the Growth Rate (or percent change) in U.S. GDP each year (from the previous year) during the Depression.

$$\text{Growth Rate} = \Delta X / X_0 = [(X_1 - X_0) / X_0] \times 100$$

$$1930 - 1931: X_0 = \$91.1 \text{ B } X_1 = \$76.4 \text{ B}$$

$$\begin{aligned} \text{Then, Growth Rate} &= [\Delta X / X_0] \times 100 \\ &= [(76.4 - 91.1) / 91.1] \times 100 \\ &= -0.161 \times 100 \\ &= -16.1\% \end{aligned}$$

Thus, there was a 16.1% drop in the U.S. GDP from 1930 to 1931.



Try the rest yourself. Can you figure out what the growth rate was between 1931 and 1932? Between 1932 and 1933?

Answers:

$$1929-30: -0.12235 \times 100 = -12.235\%$$

$$1930-31: -0.16136 \times 100 = -16.136\%$$

$$1931-32: -0.23298 \times 100 = -23.298\%$$

$$1932-33: -0.04096 \times 100 = -4.096\%$$

**NOTE (percentage point change vs. percent change):** If  $x$  increases from 65% to 70%, this would be called a **five percentage point change**, not a 5 percent change. The percent change from 65 to 70 would be:

$$\left(\frac{70 - 65}{65}\right) \times 100 = \frac{5}{65} \times 100 = .07692 \times 100 = 7.69\%$$

Thus, the 5 *percentage point* change results in a 7.7 *percent change* in  $x$ .

### ***Midpoint Method of Percent Change***

Sometimes we want to consider both movement from  $X_0$  to  $X_1$  *and* movement from  $X_1$  to  $X_0$ . In this case, it is not clear whether  $X_0$  or  $X_1$  should be the base.

Compromise: Base = average or *midpoint* of  $X_1$  and  $X_0 = (X_1 + X_0)/2$ .

Then % change =  $\Delta X / [(X_1 + X_0)/2]$ .

This is known as the Midpoint Formula.